Karlsruhe Institute of Technology

Text Indexing

Lecture 10: Top-k Document Retrieval

Florian Kurpicz

PINGO

https://pingo.scc.kit.edu/815630

Document Listing

- similar to document retrieval (next lecture)
- get all documents containing a *phrase*

Definition: Document Listing

Given a collection of D documents $\mathcal{D} = \{d_1, d_2, \ldots, d_D\}$ containing symbols from an alphabet $\Sigma = [1, \sigma]$ and a pattern $P \in \Sigma^*$, return all $j \in [1, D]$, such that d_j contains P.

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- $d_1 = \text{ATA}$
- $d_2 = \text{TAAA}$
- $d_3 = \text{TATA}$

And for queries:

- $P = \text{TA}$ is contained in d_1, d_2, and d_3
- $P = \text{ATA}$ is contained in d_1 and d_3

Basic Concepts

Definition: Document Concatenation

Given a collection of D documents $\mathcal{D} = \{d_1, d_2, \ldots, d_D\}$ containing symbols from an alphabet $\Sigma = [1, \sigma]$ where each document ends with a special symbol $\# \notin \Sigma$, the string

$$\mathcal{C} = d_1 d_2 \ldots d_D \$$$

is called the concatenation of the documents with $\$ \notin \Sigma$ and $\$ < \# < \alpha$ for all $\alpha \in \Sigma$

- $N = |\mathcal{C}| = \sum_{i=1}^{D} |d_i|$

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- $d_1 =$ ATA
- $d_2 =$ TAAA
- $d_3 =$ TATA

Document Concatenation:

- ATA#TAAA#TATA#$

Suffix Array for Document Concatenation

- given a document concatenation $\mathcal{C}$, build the suffix array
- requires $O(n)$ time
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$$DA[i] = j \iff \sum_{k=1}^{j-1} |d_k| < SA[i] \leq \sum_{k=1}^{j} |d_k|$$

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- **PINGO** are the document and suffix array sufficient to return all documents?

Naive Document Listing

- given document concatenation $\mathcal{C}$, its suffix array SA, and document array DA
- enhance suffix array to do pattern matching in $O(|P|)$ time ⓘ only briefly discussed in lecture
- find interval in suffix array matching P
- report all documents in interval in DA
- problem: $O(|P| + N)$ query time ⓘ very bad

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- is there a better solution?
- better query time
- better (or at least equal) space requirements?

Optimal Time Document Listing (1/2) [Mut02]

Definition: Chain Array

Given document concatenation $\mathcal{C}$, its suffix array *SA*, and document array *DA*, the **chain array** *CA* is defined as

$$CA[i] = \max\{j < i \colon DA[j] = DA[i]\} \cup \{0\}$$

- chains same documents together
- find lexicographically smaller suffix of same document
- use it to report documents just once
- build RMQ data structure for CA

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- **PINGO** is the chain array with RMQs enough to list all documents in optimal time?

Optimal Time Document Listing (2/2)

- given document concatenation $\mathcal{C}$, its suffix array SA, document array DA, and chain array CA with RMQ data structure
- find interval $SA[s, e]$ as before
- report document $DA[m]$ only if $CA[m] < s$ for $m \in [s, e]$

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Lemma: Optimal Document Listing

Listing all documents containing a pattern P can be done in $O(|P| + occ)$ time

Top-k Document Retrieval for Single-Term Frequencies

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Definition: Top-k Document Retrieval

Given a collection of D documents $\mathcal{D} = \{d_1, d_2, \ldots, d_D\}$ containing symbols from an alphabet $\Sigma = [1, \sigma]$, a pattern $P \in \Sigma^*$, and a threshold k , return the top-k documents $j \in [1, D]$, such that d_j contains P most often

- retrieve *occ* distinct documents where P occurs
- determine frequency of P in each document
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- total time: $O(|P| + occ(\lg k + \lg N))$

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- *occ* can be N
- can we do better

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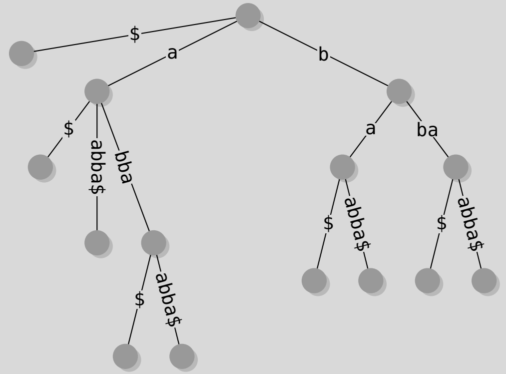
- optimal solution: $O(|P| + k)$ query time in $O(N \lg N)$ bits [NN12]
- now: $O(|P| + k \lg N)$ [GKN17]

Recap: Suffix Tree

Definition: Suffix Tree [Wei73]

A suffix tree (ST) for a text T of length n is a

- compact trie
- over $S = \{T[1..n], T[2..n], \ldots, T[n..n]\}$
  suffixes are prefix-free due to sentinel



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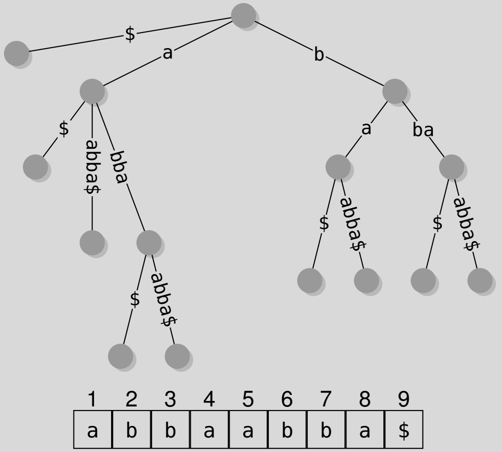
Let $G = (V, E)$ be a compact trie with root r and a node $v \in V$, then

- $\lambda(v)$ is the concatenation of labels from r to v
- $d(v) = |\lambda(v)|$ is the string-depth of v
  - string depth $\neq$ depth



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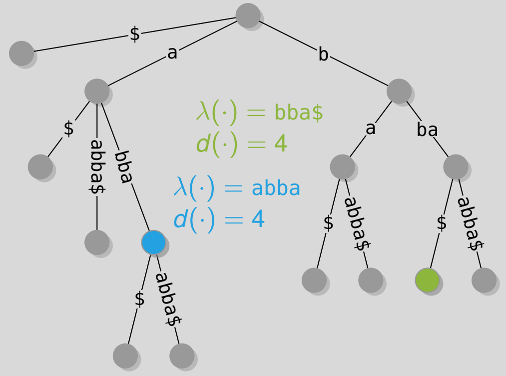

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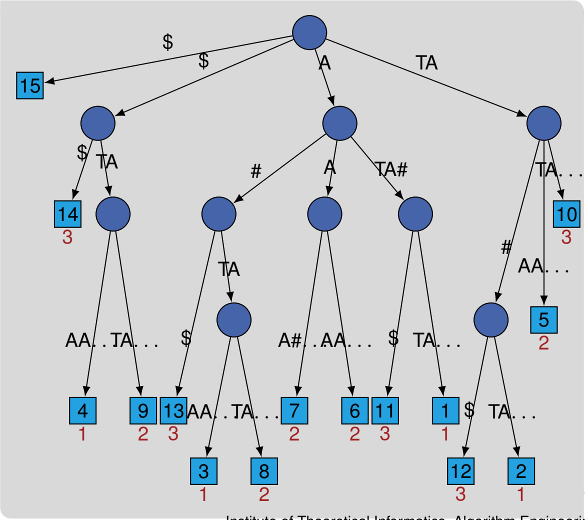
a	b	b	a	a	b	b	a	$

Generalized Suffix Tree for Top-k Document Retrieval (1/4)

- a generalized suffix tree is a suffix tree for a set of strings
- document concatenation is a set of strings

Mark Document Numbers

- mark all leaves with *DA*-entry i



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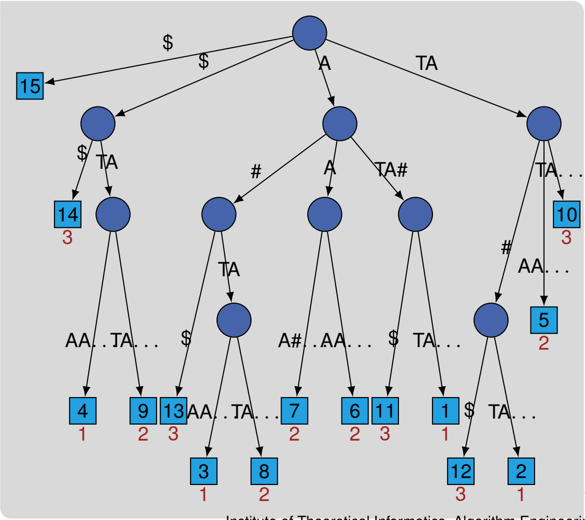
- mark all leaves with *DA*-entry i
- add i to nodes that are lowest common ancestor of two leaves marked with i



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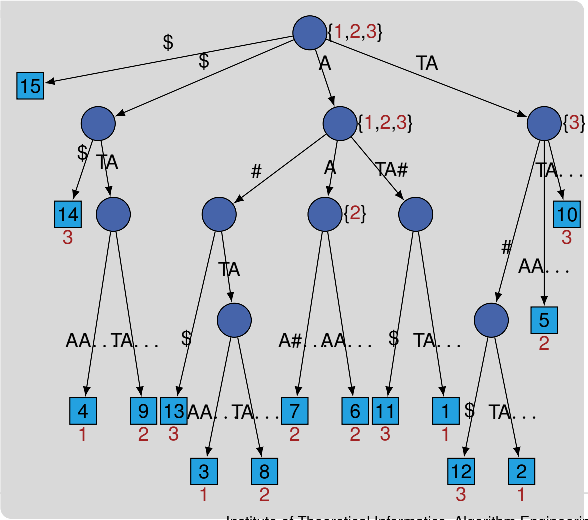


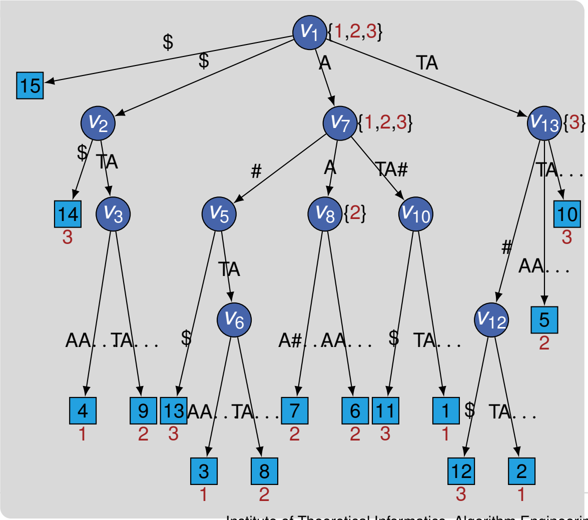
Generalized Suffix Tree for Top-k Document Retrieval (2/4)

Inner Node Names

- leaf index is rank of suffix in $[1, N]$ in leaf
- each inner node gets v gets $id(v)$, which is the leaf index of rightmost leaf in leftmost child

- $id(v) \neq id(w)$ for all inner nodes $v \neq w$
- $id(v) \in [1, N]$
- $id(v) - 1 \in [lb(v), rb(v)]$, with $[lb(v), rb(v)]$ being v's suffix array interval

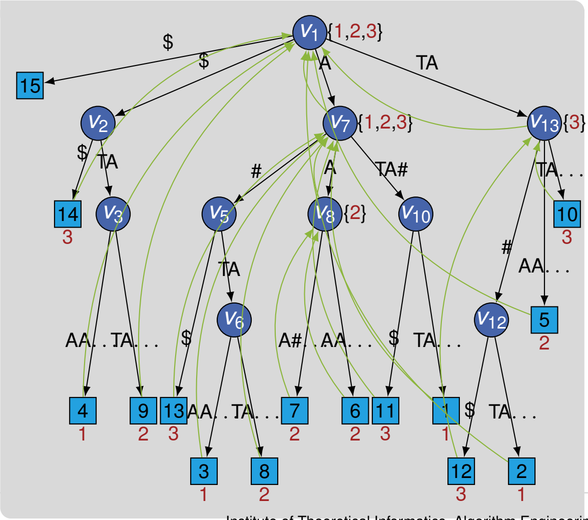
- example on the board



Generalized Suffix Tree for Top-k Document Retrieval (3/4)

- connect node with id i to closest ancestor containing id i

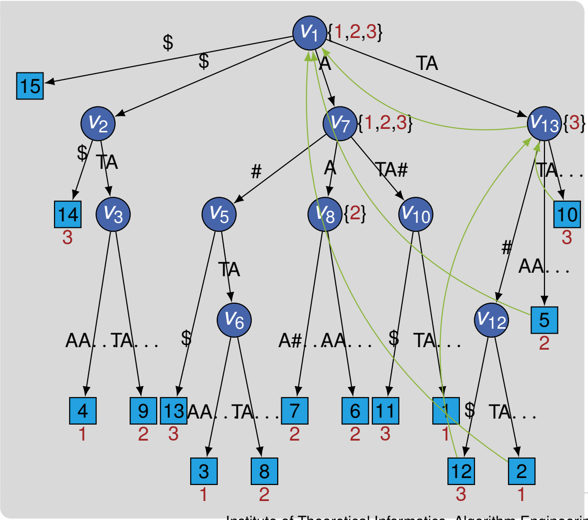
- nodes marked with id i correspond to suffix tree of d_i
- document id i occurs at most $|d_i|$ times in leaves and $|d_i| - 1$ times in inner nodes
- there are at most $O(N)$ document ids in the generalized suffix tree



Generalized Suffix Tree for Top-k Document Retrieval (4/4)

- to retrieve documents containing patter P
- select locus of P ⓘ first node v with P is prefix of $\lambda(v)$

- per document at most one pointer leaves subtree of locus v
- associate each pointer with number of occurrences of documents in pointers source (weight)
- pointer of document i leaving subtree has maximum weight of all document i pointers in subtree
- document listing is listing all documents of pointers leaving subtree

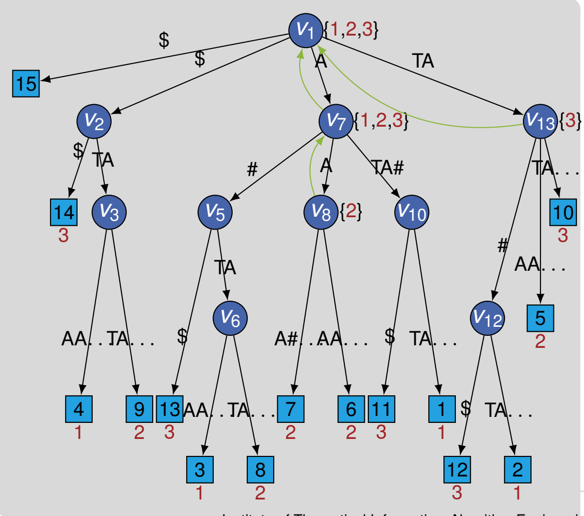


Representing Pointers on a Grid (1/2)

- now: report top-k documents
- represent pointers in a grid
- for simplicity only weights ≥ 2 starting at inner node

- assign each pointer to (x, y)-coordinate
  - x: $id(\text{source})$
  - y: $d(\text{target})$
- each point is associated with pointers weight

- given a locus v, all pointers leaving the subtree have y-coordinate $< d(v)$



Representing Pointers on a Grid (2/2)

- grid can be represented using wavelet tree
- range maximum query for each level

Answering Queries

- find string depth of locus in suffix tree
- answer range query in grid
- if represented as wavelet tree, use RMQs on each level to report top-k documents
- if $\leq k$ documents, use document listing
- total time: $O(m + k \lg N)$

Representing Pointers on a Grid (2/2)

- grid can be represented using wavelet tree
- range maximum query for each level

- **PINGO** how can we represent the pointers in a grid?

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- **PINGO** how can we represent the pointers in a grid?

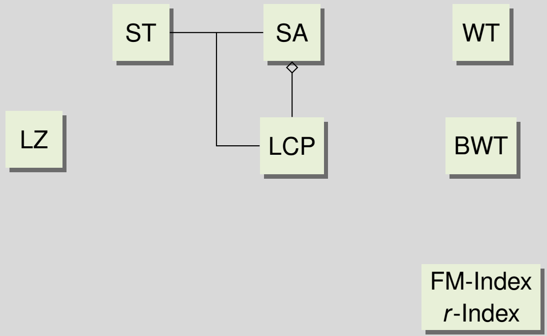
- example range queries in wavelet trees on the board

Conclusion and Outlook

This Lecture

- document listing
- top-k document retrieval (single term frequency)

Linear Time Construction



Conclusion and Outlook

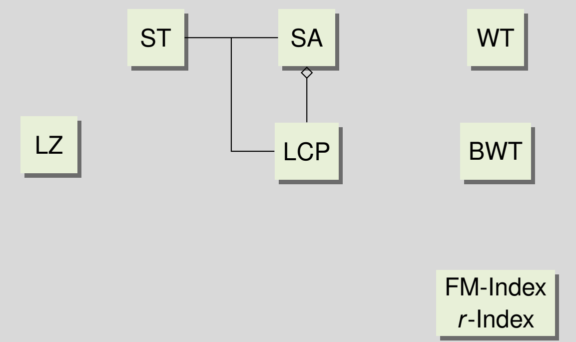
This Lecture

- document listing
- top-k document retrieval (single term frequency)

Next Lecture

- r-Index with Move data structure (finally)

Linear Time Construction



Evaluation

https://onlineumfrage.kit.edu/evasys/online.php?p=K2FFL

Bibliography I

[GKN17] Simon Gog, Roberto Konow, and Gonzalo Navarro. “Practical Compact Indexes for Top-k Document Retrieval”. In: *ACM J. Exp. Algorithmics* 22 (2017). DOI: 10.1145/3043958.

[Mut02] S. Muthukrishnan. “Efficient Algorithms for Document Retrieval Problems”. In: *SODA*. ACM/SIAM, 2002, pages 657–666.

[NN12] Gonzalo Navarro and Yakov Nekrich. “Top-k document retrieval in optimal time and linear space”. In: *SODA*. SIAM, 2012, pages 1066–1077. DOI: 10.1137/1.9781611973099.84.

[Wei73] Peter Weiner. “Linear Pattern Matching Algorithms”. In: *SWAT (FOCS)*. IEEE Computer Society, 1973, pages 1–11. DOI: 10.1109/SWAT.1973.13.