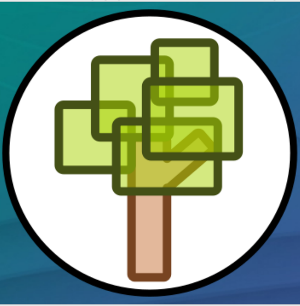
Karlsruhe Institute of Technology

# Faster Block Tree Construction

## 31st European Symposium on Algorithms (ESA 2023)

Dominik Köppl, *Florian Kurpicz*, and Daniel Meyer



# Motivation

- Git repositories
- DNA
- proteins
- user generated content W
- XML data
- book collections
- . . .

highly repetitive
&
huge input

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→ ZIP

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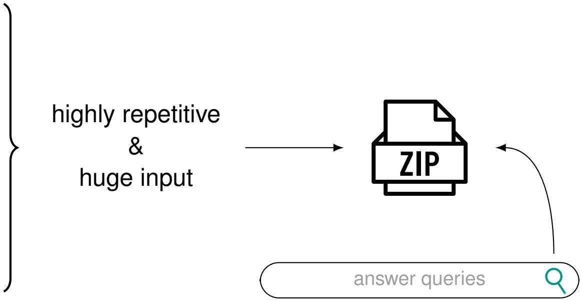

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answer queries

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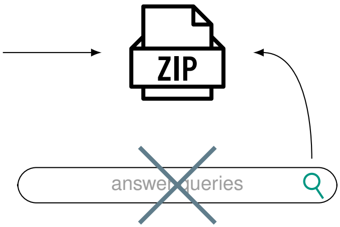

~~answer queries~~

# Compressed Self-Indices

highly repetitive
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huge input

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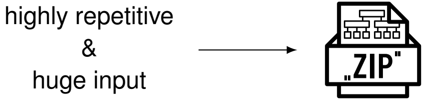

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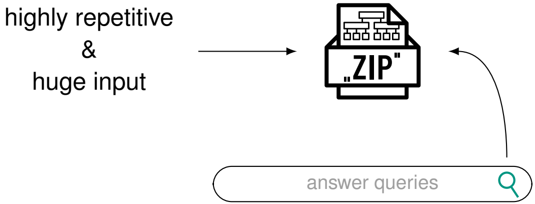

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answer queries

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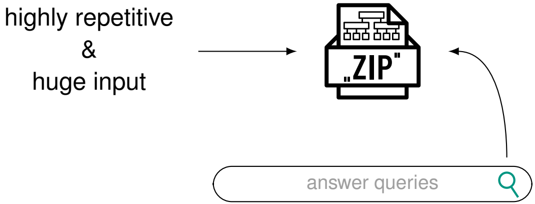

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- access
- rank
- select
- . . .
- pattern matching

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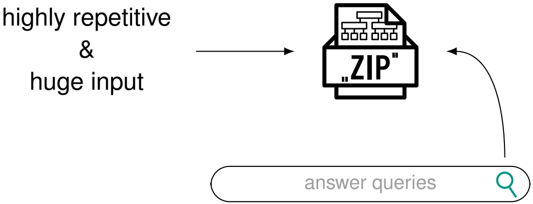
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answer queries

- access
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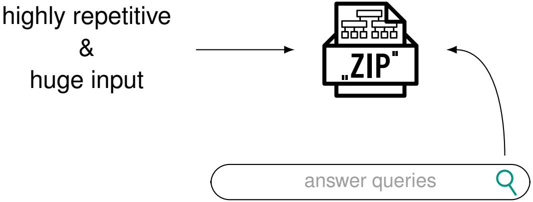
**Wavelet Tree** (de-facto standard in practice)

- $T \in [1, \sigma]^n$
- access, rank, select: $O(\log \sigma)$ time
- $nH_0(T) + o(n)$ bits space

# Compressed Self-Indices



- access
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**Wavelet Tree** (de-facto standard in practice)

- $T \in [1, \sigma]^n$
- access, rank, select: $O(\log \sigma)$ time
- $nH_0(T) + o(n)$ bits space
- **blind for repetitions**

# Measures of Repetitiveness (Excerpt)

*b* < *z* < *g* < *r* < *e*

· # phrases in bidirectional macro scheme: *b*

| a | b | r | a | c | a | d | a | b | r | a |
|---|---|---|---|---|---|---|---|---|---|---|

(6,2)r(11,1)cad(1,4)

NP-hard

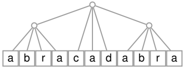
· # phrases in Lempel-Ziv parse: *z*

| a | b | r | a | c | a | d | a | b | r | a |
|---|---|---|---|---|---|---|---|---|---|---|

abr(1,1)c(1,1)d(1,4)

$O(n)$ time

· size of smallest context free grammar: *g*



NP-hard*

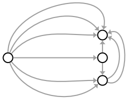
· # of runs in Burrows-Wheeler transform: *r*

| a | r | d | $ | r | c | a | a | a | a | b | b |
|---|---|---|---|---|---|---|---|---|---|---|---|

(1,a)(1,r)(1,d)(1,$)(1,r)(1,c)(4,a)(2,b)

$O(n)$ time

· # of nodes and edges in CDAWG: *e*



$O(n)$ time

* there are good heuristics

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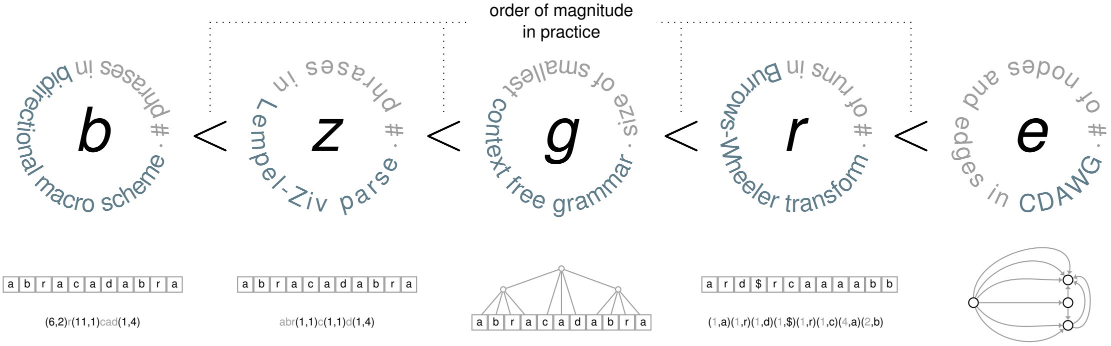
order of magnitude in practice
*b*
# phrases in bidirectional macro scheme
<
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# of nodes and edges in CDAWG
a b r a c a d a b r a
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NP-hard

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NP-hard*

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# Measures of Repetitiveness (Excerpt)

order of magnitude in practice

$b < z < g < r < e$

**$b$**: # · phrases in bidirectional macro scheme

| a | b | r | a | c | a | d | a | b | r | a |
|---|---|---|---|---|---|---|---|---|---|---|

(6,2)r(11,1)cad(1,4)

NP-hard

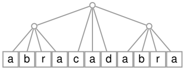
**$z$**: # · phrases in Lempel-Ziv parse

| a | b | r | a | c | a | d | a | b | r | a |
|---|---|---|---|---|---|---|---|---|---|---|

abr(1,1)c(1,1)d(1,4)

$O(n)$ time

**$g$**: size of smallest context free grammar ·



NP-hard*

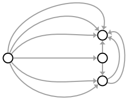
**$r$**: # · of runs in Burrows-Wheeler transform

| a | r | d | $ | r | c | a | a | a | a | b | b |
|---|---|---|---|---|---|---|---|---|---|---|---|

(1,a)(1,r)(1,d)(1,$)(1,r)(1,c)(4,a)(2,b)

$O(n)$ time

**$e$**: # · of nodes and edges in CDAWG



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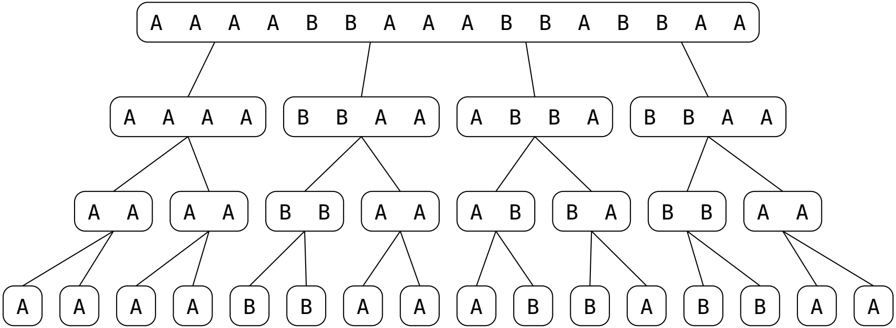
# The Block Tree



- root has degree $s = 4$
- other nodes have degree $\tau = 2$
- all levels (except the first) have $\leq 3z\tau$ blocks

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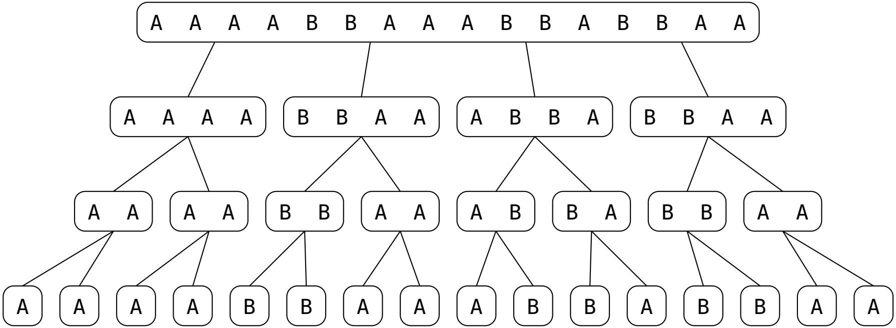

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- keep $B_i$ if consecutive $B_{i-1} \cdot B_i$ or $B_i \cdot B_{i+1}$ are leftmost occurrences

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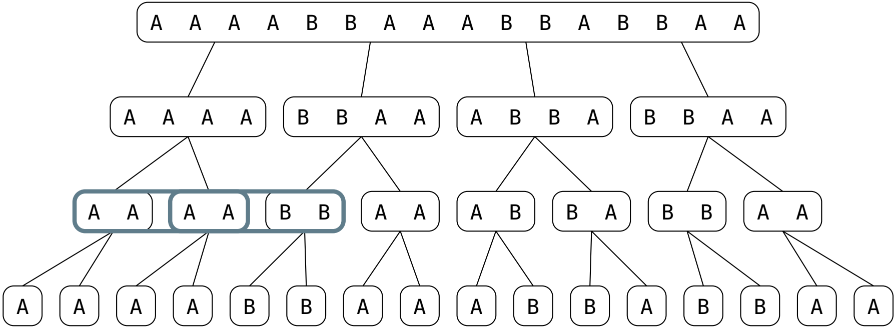

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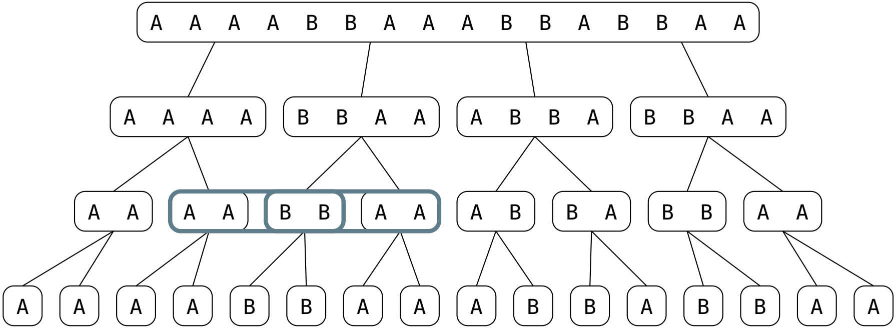

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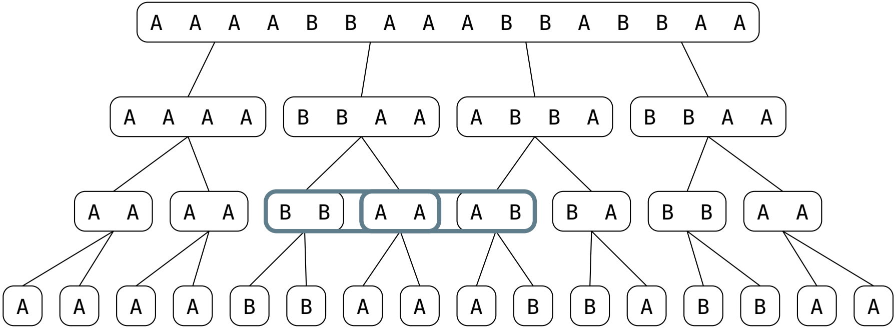

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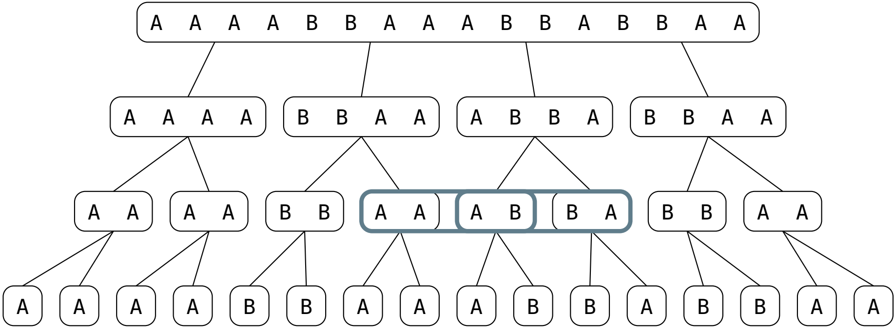

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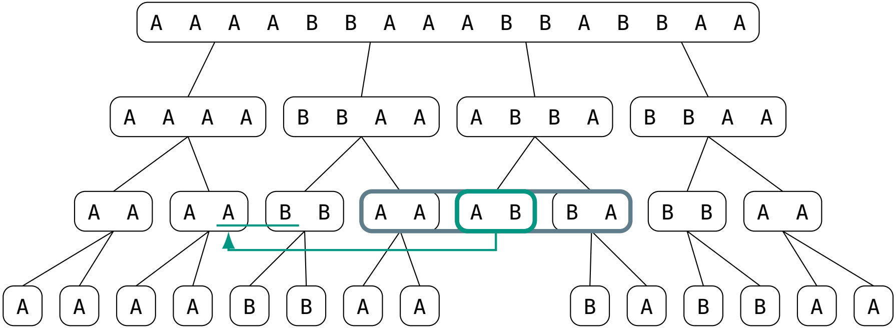

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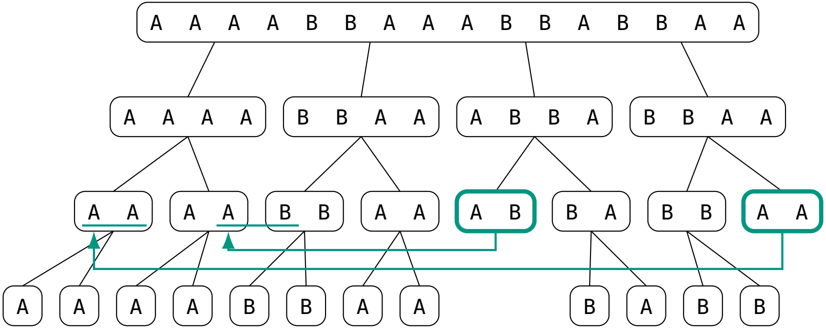

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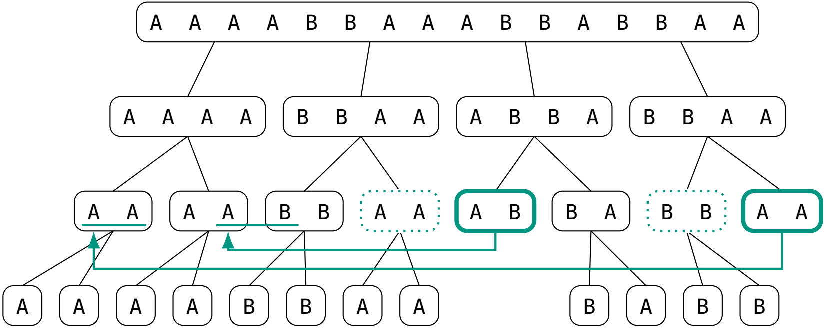

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- all levels (except the first) have $\leq 3z\tau$ blocks
- keep $B_i$ if consecutive $B_{i-1} \cdot B_i$ or $B_i \cdot B_{i+1}$ are leftmost occurrences
- remove more blocks with pruning

# State-of-Block-Tree-Construction

| Method | Reference | Working Space | Time | Implementation |
|---|---|---|---|---|
| Aho-Corasic | [Bel+21] | $O(n)$ | $O(n(1+\log_\tau(z\tau/s)))$ | no |
| Fingerprints | [Bel+21] | $O(s+z\tau\log_\tau(\frac{n\log\sigma}{s\log n}))$ | $O(n(1+\log_\tau(z\tau/s)))$ expected | yes (slow) |
| LPF Array | [here] | $O(n)$ | $O(n(1+\log_\tau(z\tau/s)))$ | yes (fast) |

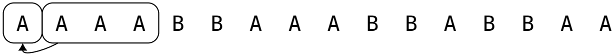
# Lempel-Ziv Parse

A A A A B B A A A B B A B B A A

# Lempel-Ziv Parse

$\boxed{\text{A}}$ A A A B B A A A B B A B B A A

# Lempel-Ziv Parse



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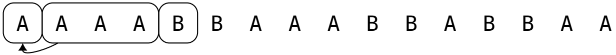

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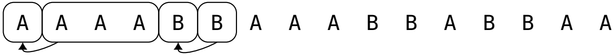

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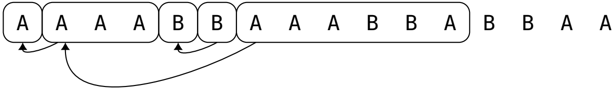

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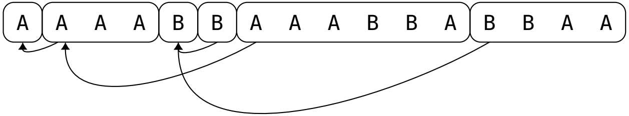

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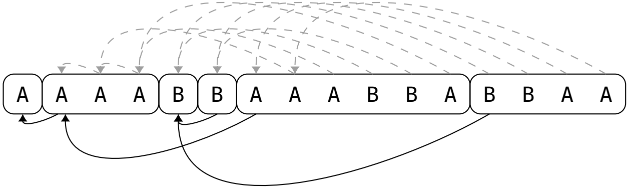

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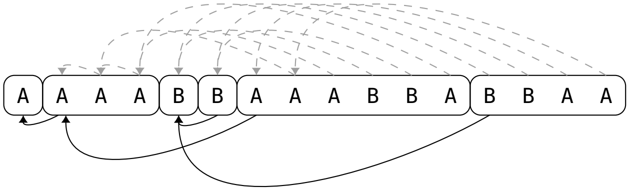
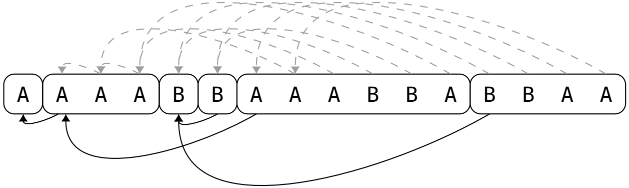


|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  | A | A | A | A | B | B | A | A | A | B | B | A | B | B | A | A |
| LPF | 0 | 3 | 2 | 1 | 0 | 1 | 6 | 5 | 4 | 3 | 2 | 5 | 4 | 3 | 2 | 1 |

# Lempel-Ziv Parse



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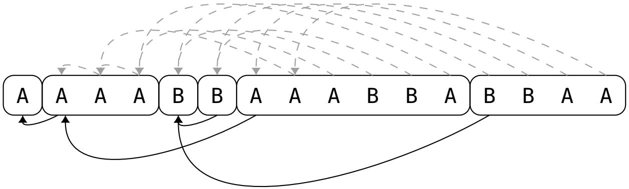
| LPF | 0 | 3 | 2 | 1 | 0 | 1 | 6 | 5 | 4 | 3 | 2 | 5 | 4 | 3 | 2 | 1 |
| PrevOcc | -1 | 1 | 2 | 3 | -1 | 5 | 2 | 3 | 4 | 5 | 6 | 4 | 5 | 6 | 7 | 8 |

# Our Algorithm (Marking of Nodes)



| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
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A A　A A　B B　A A　A B　B A　B B　A A

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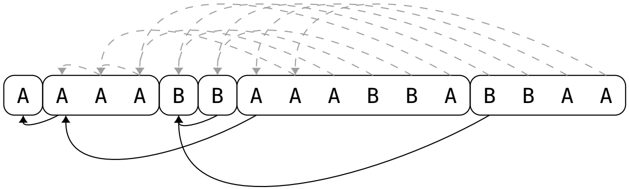

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A A A A | B B | A A | A B | B A | B B | A A

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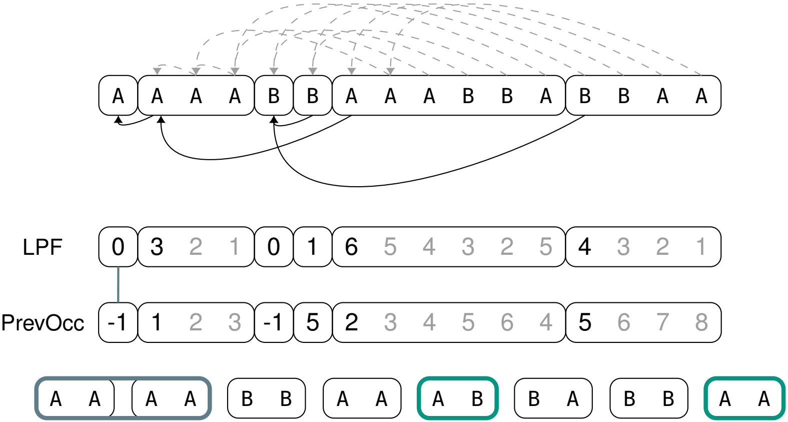

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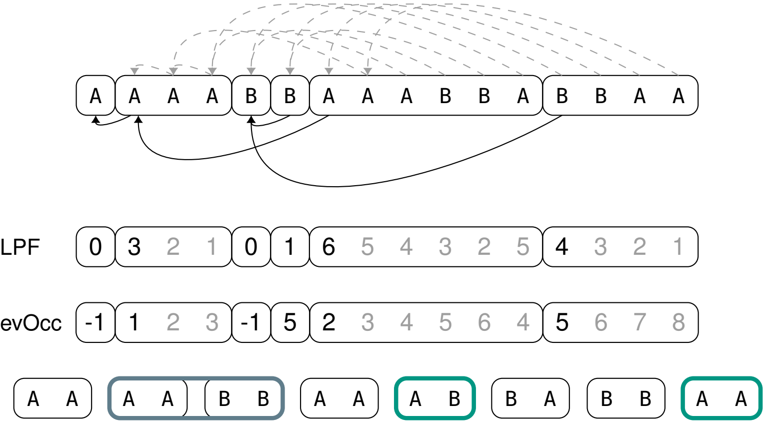

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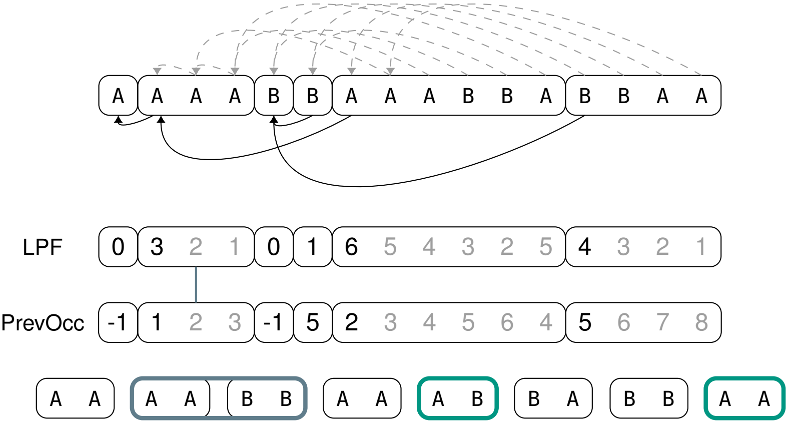

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
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A A   A A B B   A A   A B   B A   B B   A A

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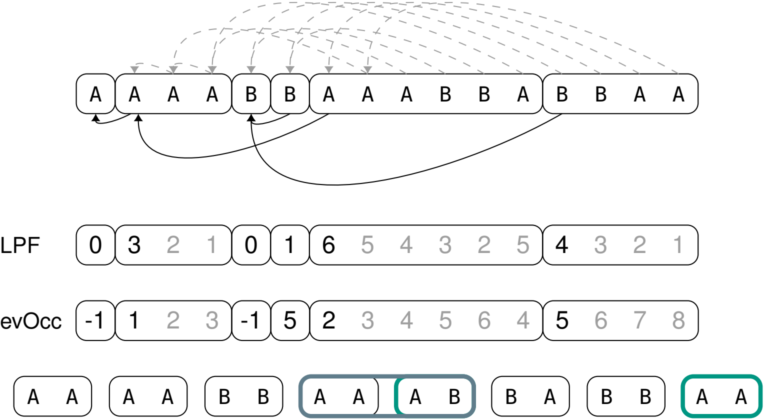

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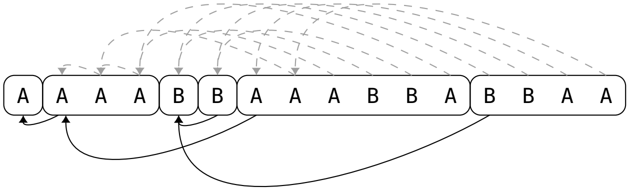

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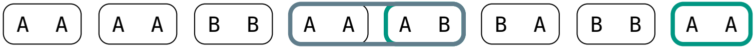

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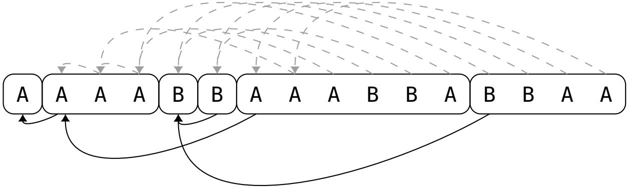
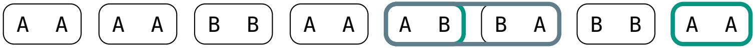
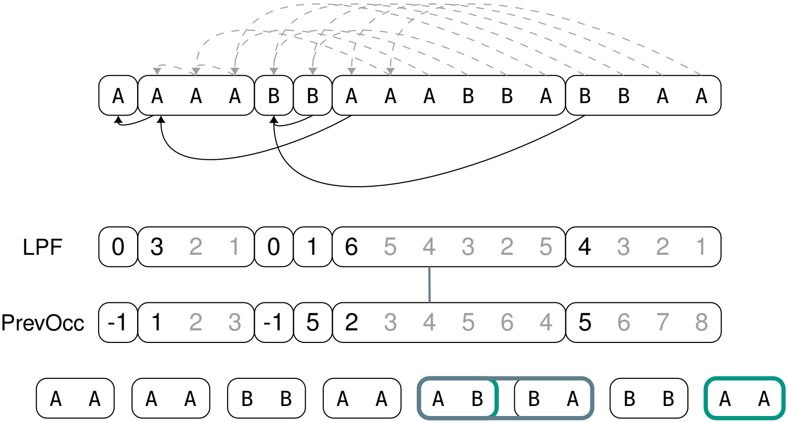


| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LPF | 0 | 3 | 2 | 1 | 0 | 1 | 6 | 5 | 4 | 3 | 2 | 5 | 4 | 3 | 2 | 1 |
| PrevOcc | -1 | 1 | 2 | 3 | -1 | 5 | 2 | 3 | 4 | 5 | 6 | 4 | 5 | 6 | 7 | 8 |



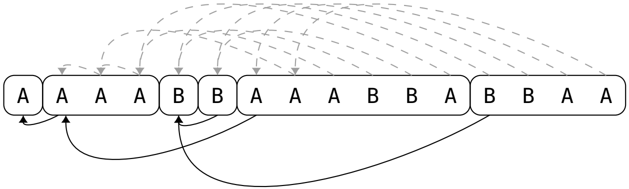
# Our Algorithm (Marking of Nodes)



| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | A | A | A | B | B | A | A | A | B | B | A | B | B | A | A |
| LPF | 0 | 3 | 2 | 1 | 0 | 1 | 6 | 5 | 4 | 3 | 2 | 5 | 4 | 3 | 2 | 1 |
| PrevOcc | -1 | 1 | 2 | 3 | -1 | 5 | 2 | 3 | 4 | 5 | 6 | 4 | 5 | 6 | 7 | 8 |

A A | A A | B B | A A | A B | B A | B B | A A

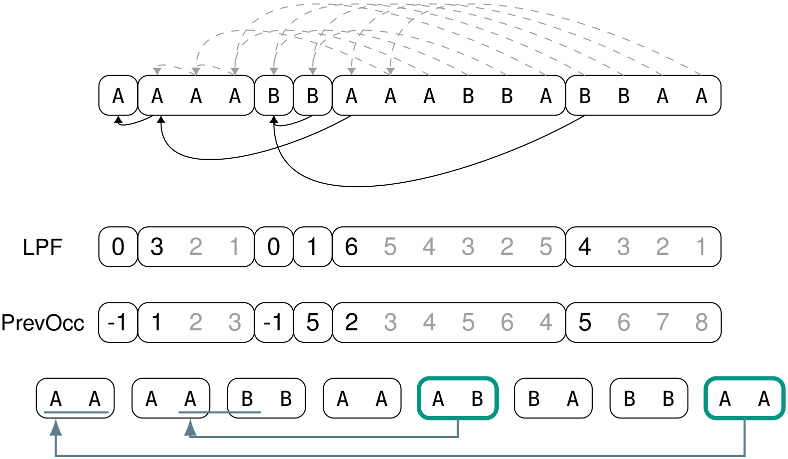
# Our Algorithm (Marking of Nodes)



| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | A | A | A | B | B | A | A | A | B | B | A | B | B | A | A |
| LPF | 0 | 3 | 2 | 1 | 0 | 1 | 6 | 5 | 4 | 3 | 2 | 5 | 4 | 3 | 2 | 1 |
| PrevOcc | -1 | 1 | 2 | 3 | -1 | 5 | 2 | 3 | 4 | 5 | 6 | 4 | 5 | 6 | 7 | 8 |

A A　A A　B B　A A　A B　B A　B B　A A

# Our Algorithm (Marking of Nodes)



|  | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  | A | A | A | A | B | B | A | A | A | B | B | A | B | B | A | A |
| LPF | 0 | 3 | 2 | 1 | 0 | 1 | 6 | 5 | 4 | 3 | 2 | 5 | 4 | 3 | 2 | 1 |
| PrevOcc | -1 | 1 | 2 | 3 | -1 | 5 | 2 | 3 | 4 | 5 | 6 | 4 | 5 | 6 | 7 | 8 |

# Experimental Evaluation

- highly tuned implementation
- tree consists only of bit and compact vectors
- tuning parameter
  - degree root $s = \{1, z\}$ (only we have $s = z$)
  - degree other nodes $\tau = \{2, 4, 8, 16\}$
  - number characters in leaves $b = \{2, 4, 8, 16\}$

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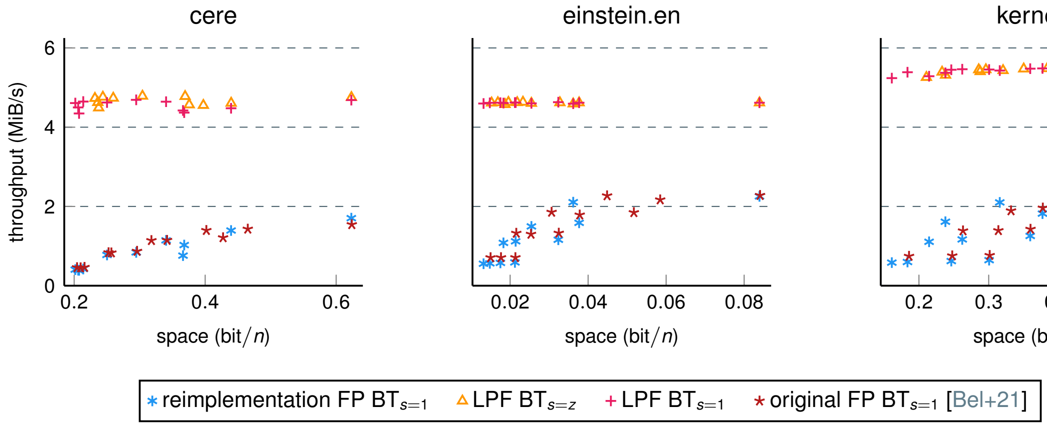
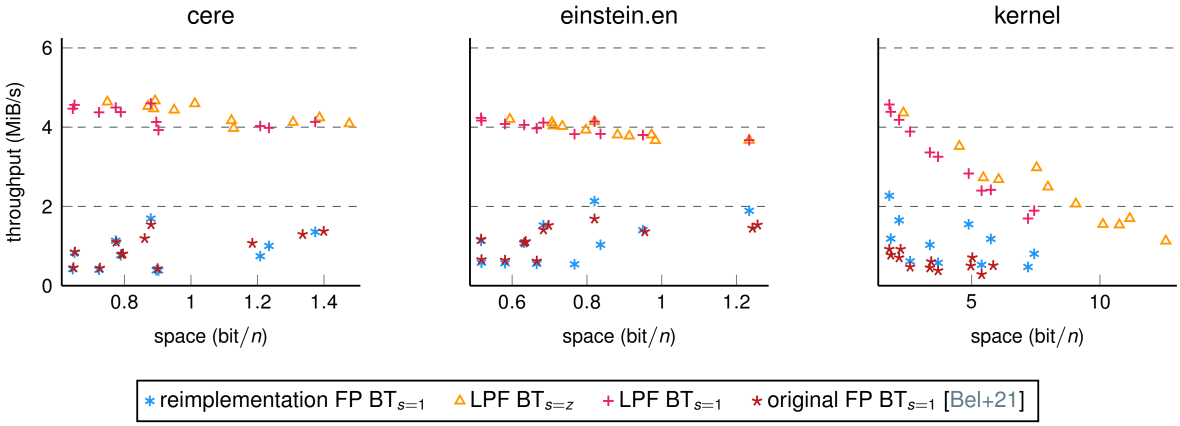
- original FP BT [Bel+21]
- our reimplementation of the original FP BT
- our LPF BT construction with $s = 1$ and $s = z$
- dynamic programming variants
- parallelization
- no comparison with wavelet trees (faster)

- repetitive instances from P&C corpus
- non-repetitive instances from P&C corpus

# Highly Repetitive Inputs (Access Only)



# Highly Repetitive Inputs (with Rank and Select Support)



# Conclusion and Future Work

- fastest block tree construction algorithm
- first parallel block tree construction
- works in practice for non-repetitive inputs

- better scaling parallel construction
- better marking rules (less pruning)

/pasta-toolbox/block_tree

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Established by the European Commission

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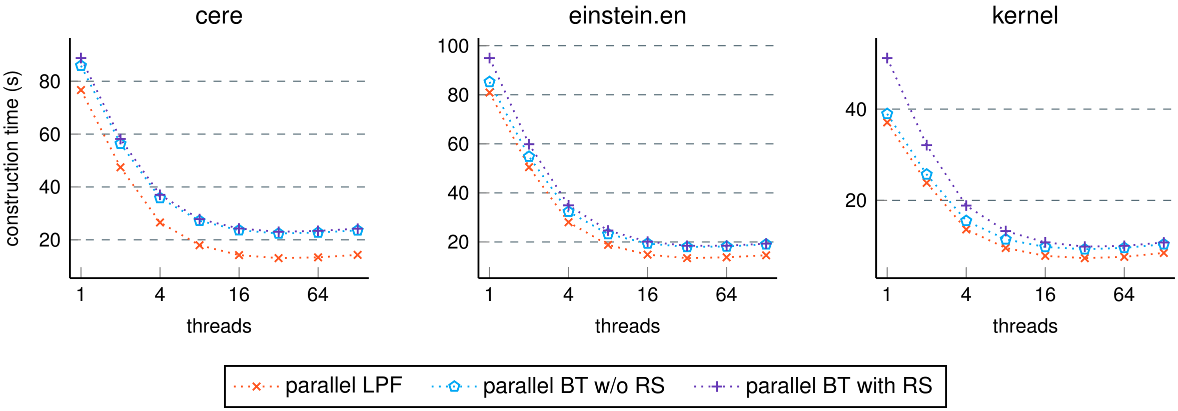
# Parallel Construction (Strong Scaling)



# Bibliography

[Bel+21] Djamal Belazzougui, Manuel Cáceres, Travis Gagie, Pawel Gawrychowski, Juha Kärkkäinen, Gonzalo Navarro, Alberto Ordóñez Pereira, Simon J. Puglisi, and Yasuo Tabei. “Block Trees”. In: *J. Comput. Syst. Sci.* 117 (2021), pages 1–22. DOI: 10.1016/j.jcss.2020.11.002.